RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2015

SECOND YEAR

Date : 26/05/2015 **MATHEMATICS FOR ECO (General)**

Time : 11 am – 2 pm **Paper :** IV Full Marks : 75

[Use separate Answer Book for each group]

Group – A

Answer ***any six*** questions : (6 × 5)

1. (a) Show that every subgroup of a cyclic group is normal. 2

   (b) Let $T:\vee \to \vee$ be an invertible Linear Transformation then show that $T^{-1}:\vee \to \vee$ is also a linear transformation. 3

2. (a) Prove that $\langle G,\cdot \rangle$ is an abelian group if $(a-b)^2 = a^2b^2,\ \forall a,b \in G$. 2

   (b) Let $T:\mathbb{P}_2(x) \to \mathbb{P}_2(x)$ be a linear transformation where $\mathbb{P}_2(x)$ is the space of all real polynomials with degree at most 2. 3

   Let, $T:\mathbb{P}_2(x) \to \mathbb{P}_2(x)$ be defined by, $T\ \ f(x)\ = \dfrac{d}{dx}\ f(x)$. Then find the kernel of *T*.

3. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show for every integer $n \geq 3$, that $A^n = A^{n-2} + A^2 - I$ Hence determine A^{50} . 3+2

4. Write the associated matrix *A* of the quadratic form $Q = 6x^2 + 35y^2 + 11z^2 + 4zx$. Find the eigen values of *A* & hence determine the value class of *Q*. 5

5. Diagonalize the matrix $A = \begin{bmatrix} 2 & 6 \\ 0 & -1 \end{bmatrix}$. 5

6. (a) Define Invariant subspace of $\vee$ under the linear Transformation $T:\vee \to \vee$. 3

   (b) Define the similarity of two matrices. 2

7. (a) Define Convex hull. 1

   (b) Prove that in E^2, the set: $X =\ \ (x,y)/\, x^2 + y^2 \leq 4$ is a convex set. 4

8. The matrix of a Linear Transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ relative to the ordered basis

   $(-1,1,1),(1,-1,1),(1,1,-1)$ of $\mathbb{R}^3$ is $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{pmatrix}$.

   Find the Linear Transformation. 5

9. Let $\alpha_1,\alpha_2,\alpha_3$ be an ordered basis of a real vector space $\vee$ and a linear transformation $T:\vee \to \vee$ is defined by $T(\alpha_1) = \alpha_1 + \alpha_2 + \alpha_3,\ T(\alpha_2) = \alpha_1 + \alpha_2,\ T(\alpha_3) = \alpha_1$. Show that *T* is non-singular. Find the matrix of T^{-1} relative to the ordered basis $\alpha_1,\alpha_2,\alpha_3$. 5

Group – B

10. Answer ***any one*** question : (1 × 5)

    (a) (i) A businessman has the option of investing his money in two plans. Plan *A* guarantees that each rupee invested will earn seventy paise a year hence, while plan *B* guarantees that each rupee invested will earn two rupees two years hence. In plan *B*, only investments for periods that are multiples of two years are allowed. The problem is how should he invest ten thousand rupees in order to maximize the earnings at the end of three years.

Formulate this problem as a linear programming model. 4

(ii) What do you mean by linearly independent vectors? 1

(b) Solve the following linear programming problem graphically.

Maximize $z = 2x_1 + 3x_2$

Subject to $x_1 - x_2 \leq 1$

$x_1 \leq 3$

and $x_1, x_2 \geq 0$ 5

11. Answer ***any two*** questions : (2 × 10)

(a) (i) Reduce the following linear programming problem to standard maximization form: 4

Minimize $z = 2x_1 - 3x_2 + 6x_3$

Subject to $x_1 + x_2 - x_3 \geq -6$

$-6x_1 + 7x_2 + 4x_3 = 15$

$|13x_1 - 4x_2 + 5x_3| \leq 13,$

$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

(ii) Find the dual of the following linear programming problem: 6

Minimize $z = 3x_1 - 2x_2 + 4x_3$

Subject to $3x_1 + 5x_2 + 4x_3 \geq 7,$

$6x_1 + x_2 + 3x_3 \geq 4,$

$7x_1 - 2x_2 - x_3 \leq 10,$

$x_1 - 2x_2 + 5x_3 \geq 3,$

$4x_1 + 7x_2 - 2x_3 \geq 2,$

$x_1, x_2, x_3 \geq 0$

(b) (i) Find the extreme points of the convex set of the feasible solution of the following L.P.P. 5

Minimize $z = 2x_1 + 3x_2 + 4x_3 + 5x_4$

Subject to $2x_1 + 3x_2 + 5x_3 + 6x_4 = 16,$

$x_1 + 2x_2 + 2x_3 + 3x_4 = 9,$

$x_1, x_2, x_3, x_4 \geq 0$

Also find the optimal basic feasible solution for given problem.

(ii) Solve the following L.P.P by Big-M method. 5

Maximize $z = -2x_1 + x_2 + 3x_3$

Subject to $x_1 - 2x_2 + 3x_3 = 2,$

$3x_1 + 2x_2 + 4x_3 = 1,$

and $x_1, x_2, x_3 \geq 0$

(c) (i) Find the optimal assignment for the assignment problem with the following cost matrix. 5

	1	2	3	4
A	10	12	18	11
B	5	6	7	8
C	12	14	13	11
D	8	6	11	9

(ii) Find the dual of the following L.P.P. 5

Maximize $z = 2x + 3y + 5z$

Subject to
$$2x - y + 3z \le 1$$
$$4x + 3y - 7z \le 5$$
$$3x + 2y + z \le 2$$
$$x, y, z \ge 0$$

Group – C

12. Answer **any two** questions : {2×5}

a) Consider the payoff of the following two person zero sum game— 5

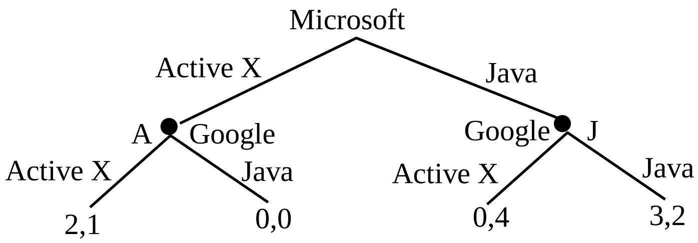
		Player B	
		Strategy 1	Strategy 2
Player A	Strategy 1	7	6
	Strategy 2	5	8

Find out the equilibrium strategy. Does it represents the saddle point?

b) Find out equilibrium through iterated elimination of strictly dominated strategies from the following : 5

	L	C	H
U	0,2	3,1	2,3
M	1,4	2,1	4,1
D	2,1	4,4	3,2

c) Find out the equilibrium through the method of backward induction— 5



13. Answer **any one** question : (1×10)

a) Show that there is always a Nash Equilibrium in a two person zero sum game in mixed strategies. 10

b) Distinguish between a static game and a dynamic game. Show with an example how the dynamic game solves the problem of multiple equilibrium found in static games. 10

——————— × ———————